Discontinuous approximation of flow in porous media with adsorption

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Polymer flooding in petroleum reservoir engineering can be described by a Brinkman-based model of flow in porous media coupled to a non-strictly hyperbolic system of conservation laws for multiple components that form the aqueous phase. The coupled flow-transport problem is discretized by coupling an $\mathbf{H}(\mathrm{div})$-conforming discontinuous Galerkin (DG) method for the Brinkman flow problem with a classical DG method for the transport equations.

1 Coupled flow-transport model

Polymer flooding as a process within petroleum reservoir engineering can be described as multicomponent viscous flow in porous media with adsorption. Specifically, we consider a system of conservation laws for the water and polymer components that form the aqueous phase whose saturation is s, and where t is time, φ is the (constant) rock porosity, and the flux vector $\boldsymbol{F}$ depends on s, the concentrations of polymers $\boldsymbol{c} := (c_1, \ldots, c_{\mathcal{M}})^{\mathrm{T}}$, the volume average flow velocity $\boldsymbol{u}$, and spatial position $\boldsymbol{x}$:

$$\begin{aligned} &\varphi \partial_t s + \operatorname{div} \boldsymbol{F} = 0, \quad \text{where} \quad \boldsymbol{F} = f(s,\boldsymbol{c})\boldsymbol{u} + b(s,\boldsymbol{c})\boldsymbol{K}(\boldsymbol{x})\boldsymbol{g}, \quad f(s,\boldsymbol{c}) := \lambda_{\mathrm{w}}(s,\boldsymbol{c})/(\lambda_{\mathrm{w}}(s,\boldsymbol{c}) + \lambda_{\mathrm{n}}(s)), \\ &b(s,\boldsymbol{c}) := f(s,\boldsymbol{c})\lambda_{\mathrm{n}}(s)(\rho_{\mathrm{w}} - \rho_{\mathrm{n}}), \quad \lambda_{\mathrm{w}} = k_{\mathrm{rw}}/\mu_{\mathrm{w}}, \quad \lambda_{\mathrm{n}} = k_{\mathrm{rn}}/\mu_{\mathrm{n}}. \end{aligned} \tag{1}$$

Here w and n indicate the wetting and non-wetting phases, λ_{w} and $\lambda_{\mathrm{n}} = \mathrm{const.}$ are phase mobilities, k_{rw} and k_{rn} are phase relative permeabilities (chosen here by the Brooks-Corey model [1]), and μ_{w} and μ_{n} are phase viscosities. The viscosity μ_{w} is here given by $\mu_{\mathrm{w}}(\boldsymbol{c}) = \mu_{\mathrm{w},0} + a(c_1 + \cdots + c_{\mathcal{M}})$, where $\mu_{\mathrm{w},0}$ is the viscosity of the wetting phase and a is a positive constant.

Under the same assumptions, the transport of the polymers in the aqueous phase is described by the following continuity equations, where ρ_{r} is the density of rock and $a_l(c_l)$ is the adsorption of polymer c_l per unit mass of the rock:

$$\varphi \partial_t (s c_l) + \partial_t \big((1-\varphi)\rho_{\mathrm{r}} a_l(c_l)\big) + \operatorname{div}(c_l \boldsymbol{F}) = 0, \quad l = 1, \ldots, \mathcal{M}. \tag{2}$$

Note that (1) and (2) are coupled in s and $\boldsymbol{c}$. Moreover, $\boldsymbol{F}$ depends discontinuously on $\boldsymbol{x}$ if the porous medium is heterogeneous. Finally, $\boldsymbol{u}$ is determined from a Brinkman model that represents the momentum and mass balance of the mixture:

$$\boldsymbol{K}(\boldsymbol{x})^{-1}\boldsymbol{u} - \operatorname{div}\big(\mu(s,\boldsymbol{c})\boldsymbol{\varepsilon}(\boldsymbol{u}) - p\boldsymbol{I}\big) = (\rho_{\mathrm{w}} - \rho_{\mathrm{n}})s\boldsymbol{g}, \quad \operatorname{div}\boldsymbol{u} = j; \quad \boldsymbol{\varepsilon}(\boldsymbol{u}) := \tfrac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}), \tag{3}$$

where p is the pressure, $\boldsymbol{g}$ is the gravitational acceleration, j is a mass source or sink, $\boldsymbol{K}(\boldsymbol{x})$ is the absolute permeability tensor, and $\mu(s,\boldsymbol{c}) = 1/(\lambda_{\mathrm{w}} + \lambda_{\mathrm{n}})$. The constants ρ_{w} and ρ_{n} are the densities of aqueous and oleic phase, respectively. For an isotropic medium, $\boldsymbol{K} = \kappa(\boldsymbol{x})\boldsymbol{I}$, where κ is a scalar function (assumed uniformly bounded $\kappa_{\min} \le \kappa(\boldsymbol{x}) \le \kappa_{\max}$), and $\boldsymbol{I}$ is the 2×2 identity matrix. Thus, the sought quantities are the $\mathcal{M}+4$ scalar components of s, $\boldsymbol{c}$, $\boldsymbol{u}$ and p as functions of $\boldsymbol{x}$ and t defined by the coupled system (1)–(3) of the same numbers of scalar PDEs, plus initial and boundary conditions.

2 Discretization and numerical example

The discretization for (1)–(3) analyzed in [2] combines an existing [5] $\mathbf{H}(\mathrm{div})$-conforming discontinuous Galerkin (DG) method for the Brinkman flow problem (3) with a classical DG method for the transport equations. The DG formulation of the transport problem is based on discontinuous numerical fluxes [4]. We refer to [2] for any detail. An important novelty in the numerical approximation of (1), (2) is the use of discontinuous numerical fluxes for the coupled equations that arise for the continuous-in-time, spatially discrete equations that represent the system (1), (2). This is achieved by treating each approximate polymer concentration as a discontinuous coefficient entering the numerical flux, see [4, §2.4]. An invariant region property, stating that the average saturation on each element stays between zero and one and the average polymer concentration only assumes values between those of the average of neighboring or boundary cells, is proved in [2] under the (mild) assumption that the underlying mesh is a **B**-triangulation [3] plus a suitable time step restriction (CFL condition). Thus, only physically relevant saturation and concentration values are generated by the scheme.

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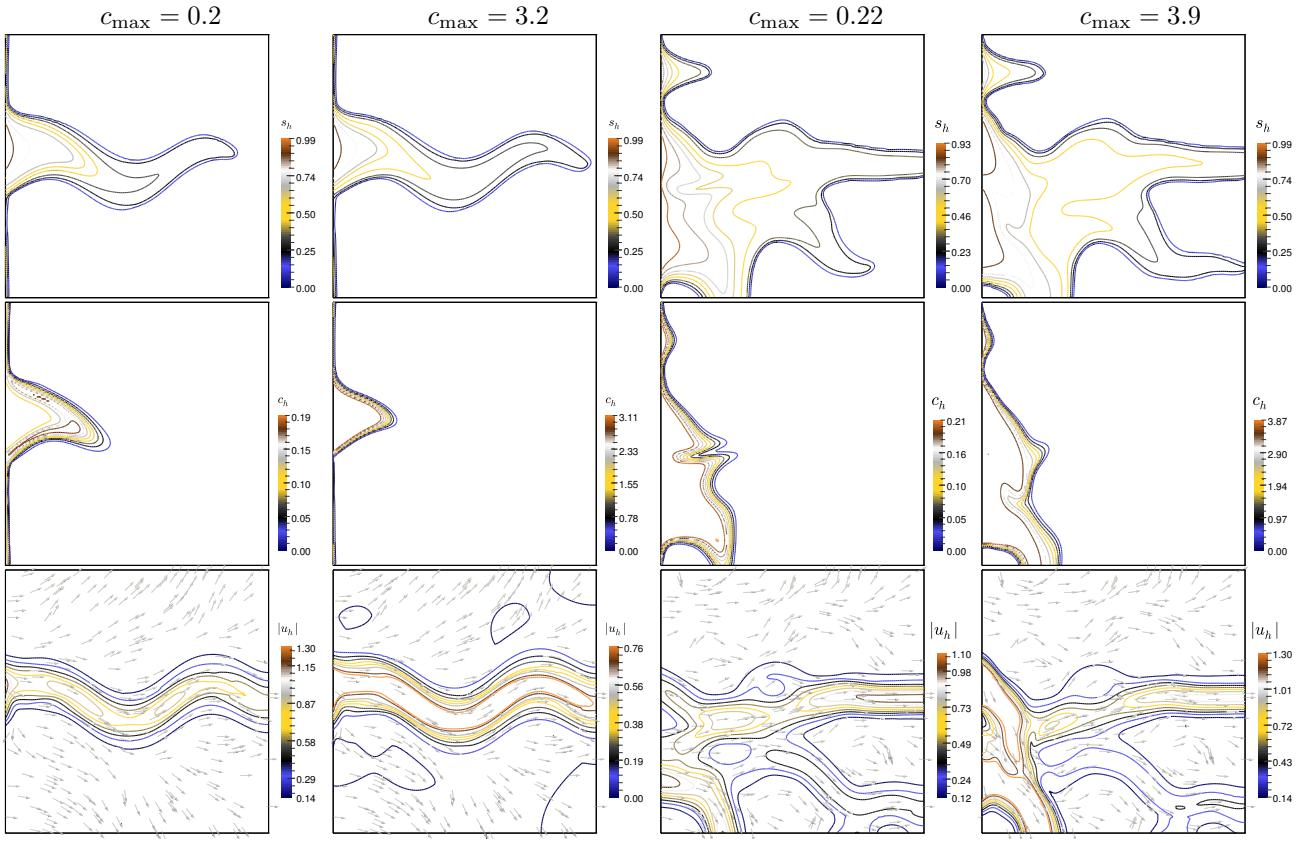

Fig. 1: Numerical simulation of polymer flooding in a reservoir, corresponding (first and second column) to $t = 0.2$ for the case of a sinusoidal winding crack and (third and fourth column) to $t = 0.25$ for randomly distributed spots of reduced permeability, showing numerical values (top) of saturation s, (middle) of polymer concentration c and (bottom) of velocity $\boldsymbol{u}$.

Numerical tests presented in [2] illustrate the accuracy and stability of the proposed method. We here present additional simulations (not included in [2]) of polymer flooding in a reservoir. We choose $\mathcal{M} = 1$ along with $a_1(\boldsymbol{c}) = a(c) = c_{\max}\frac{a_0 c}{1+a_0 c}$ (Langmuir adsorption isotherm), $\mu_{\rm w}(c) = \mu_{\rm w,0} + 0.75\frac{\mu_{\rm w,0}c}{c_{\max}}$, $\rho_{\rm w} = 1$, $\rho_{\rm n} = 0.58$, $\mu_{\rm w}^0 = 0.35$, $\mu_{\rm n} = 3.5$, $\varphi = 0.25$, $a_0 = 0.1$, $\lambda_{\rm w}(s,c) = s^2/\mu_{\rm w}(c)$, $\lambda_{\rm n} = (1-s)^2/\mu_{\rm n}$. The domain is $\Omega = (0,1)^2$. In the two left columns of Figure 1 the heterogeneity in the medium is described by the sinusoidal absolute permeability $\kappa(x,y) = \max\{\exp(-10y + 5 + \sin(10x))^2, 0.01\}$, and in the two others we utilize a permeability distribution incorporating a non-homogeneous field where 25 low-permeability disks of radius 0.005 are randomly located in Ω. More precisely, we set $\kappa(x,y) = \max\{\sum_{i=1}^{25}\exp(-200((x-q_x(i))^2 + (y-q_y(i))^2)), 0.0001\}$, where the random points are $(q_x(i), q_y(i))$. A quadratic pressure profile is imposed in the form of a natural boundary condition for the first and a linear pressure profile is imposed for the second pair of examples. The porous slab is assumed to be initially full of oil and a constant profile of water ($s = 1$) and polymers (of concentration $c_{\max}$ indicated in each case) is injected on the left wall. For the examples with distributed permeability field the rock porosity is decreased to $\varphi = 0.2$. The domain is divided into 32K triangular elements and we can observe the differences between the sweeping process (of transporting the oleic phase from the inlet boundary to the outlet) according to the inlet concentration of polymers. The numerical results imply that more oil is displaced in the presence of a higher polymer concentration, but due to gravity the polymers will tend to get retained within the reservoir (see the right columns of Figure 1).

Acknowledgements We acknowledge funding by Fondecyt project 1170473; CRHIAM, project CONICYT/FONDAP/15130015; EPSRC research grant EP/R00207X/1; and by the Dirección de Investigación y Desarrollo of Universidad de La Serena, DIDULS project PR17151.

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